

# **Non-Relativistic Quantum Mechanics**

**Lecture notes – FYS 4110**

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## **Forord**

Disse notatene er skrevet som et kompendium til kurset FYS 4110, Ikke-relativistisk kvantemekanikk høsten 2003. De er for tiden under utarbeidelse og vil bli oppdatert ettersom mer stoff blir ferdig og evt. feil blir korrigert. Jeg mottar gjerne kommentarer og korreksjoner. Jeg har valgt skrive kompendiet paa engelsk, selv om kurset foreleses paa norsk (i høst). Dette er bl.a. fordi faguttrykkene er vel saa godt innarbeidet paa engelsk, og fordi det vil være lesbart for evt. engelskspraaklige studenter paa et senere tidspunkt.

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